CBCS Scheme

USN | | | | | | | | | | **16/17MCA15**

First Semester MCA Degree Examination, Dec.2017/Jan.2018
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: ***Answer FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Let p and q be the propositions "Swimming at the New-Jersey shore is allowed" and "Sharks have been near the shore" respectively. Express each of these compound proposition as an English sentence.

i) $p \to \neg q$ ii) $\neg p \to \neg q$ iii) $p \leftrightarrow \neg q$ iv) $\neg p \wedge (p \vee \neg q)$. **(08 Marks)**

b. Show that $\neg p(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent. **(04 Marks)**

c. Determine the truth value of each of these statement if the domain for all variables consists of all integers.

i) $\forall_n(n^2 \geq 0)$ ii) $\exists_n(n^2 = 2)$ iii) $\forall_n(n^2 \geq n)$ iv) $\exists_n(n^2 < 0)$. **(04 Marks)**

OR

2 a. Show that the following argument is valid.

$$\begin{array}{l} (\neg p \vee \neg q) \to (r \wedge s) \\ r \to t \\ \neg t \\ \hline \therefore p \end{array}$$

(06 Marks)

b. Show that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam". Imply the conclusion "Someone who passed the first exam has not read the book". **(06 Marks)**

c. Prove the following statement in direct and contradiction method : If n is an odd integer then n + 11 is even. **(04 Marks)**

Module-2

3 a. Provide the proof, for any universe U, let $A \subseteq U_1$ then $\phi \subseteq A$, and if $A \neq \phi$, then $\phi \subset A$. **(06 Marks)**

b. Let A, B and C be sets, show that : $\overline{A \cup (B \cap C)} = (\overline{C} \cup \overline{B}) \cap \overline{A}$. **(04 Marks)**

c. Let f_1 and f_2 be functions from R to R such that $f_1(x) = x^2$ and $f_2(x) = x - x^2$. What are the functions $f_1 + f_2$ and $f_1 \cdot f_2$? **(06 Marks)**

OR

4 a. Suppose that g is a function from A to B and f is a function from B to C.
   i) Show that if f and g are one – to – one functions then f o g is also one – to – one.
   ii) Show that if both f and g are onto functions then f o g is also onto. **(04 Marks)**

b. Let N is the set of natural numbers. The relation R is defined on $N \times N$ as follows : $(a, b) R(c, d) \Leftrightarrow a + d = b + c$. Prove that R is an equivalence relation. **(06 Marks)**

c. Suppose A and B are ordered sets. Show that the product order on $A \times B$ defined by $(a, b) \leq (c, d)$ if $a \leq c$ and $b \leq d$ is a partial order on $A \times B$. **(06 Marks)**

Module-3

5 a. During a local campaign, eight Republican and five Democratic candidates are nominated for president of the school board.
   i) If the president is to be one of these candidates, how many possibilities are there for the eventual winner?
   ii) How many possibilities exist for a pair of candidates (one from each party to oppose each other for the eventual election)? **(04 Marks)**

b. Let A_1 and A_2 be finite sets, prove that :
   $|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2|$ **(06 Marks)**

c. State and prove the pigeonhole principle. **(06 Marks)**

OR

6 a. i) How many permutations of {a, b, c, d, e, f, g} end with a?
   ii) In how many ways can a set of five letters be selected from the English alphabet? **(04 Marks)**

b. State and prove the Binomial theorem. **(06 Marks)**

c. Find the recurrence relation for the Tower of Hanoi problem. **(06 Marks)**

Module-4

7 a. Two integers are selected, at random and without replacement from {1, 2, 3, ……, 99, 100}. What is the probability their sum is even? **(04 Marks)**

b. If you toss a fair coin four times, what is the probability that you will get two heads and two tails? **(04 Marks)**

c. An integer is selected at random from 3 through 17 inclusive. If A is the event that a number divisible by 3 is chosen and B is the event that the number exceeds 10, determine :
   $P_r(A)$, $P_r(B)$, $P_r(A \cap B)$, and $P_r(A \cup B)$. How is $P_r(A \cup B)$ related to $P_r(A)$, $P_r(B)$, and $P_r(A \cap B)$? **(08 Marks)**

OR

8 a. Let S be the sample space for an experiment E. For events A, B, $C \leq S$,
$P_r(A \cup B \cup C) = P_r(A) + P_r(B) + P_r(C) - P_r(A \cap B) - P_r(A \cap C) - P_r(B \cap C) + P_r(A \cap B \cap C)$. **(08 Marks)**

b. A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French and Russian, how many students have taken a course in all three languages? **(08 Marks)**

Module-5

9 a. Prove that an undirected graph G = (V, E) has an even number of vertices of odd degree. **(06 Marks)**

b. Draw these graphs : **(04 Marks)**
i) K_7 ii) $K_{2,8}$.

c. Show that the graph G = (V, E) and H = (W, F) are isomorphic.(Refer Fig.Q9(c)). **(06 Marks)**

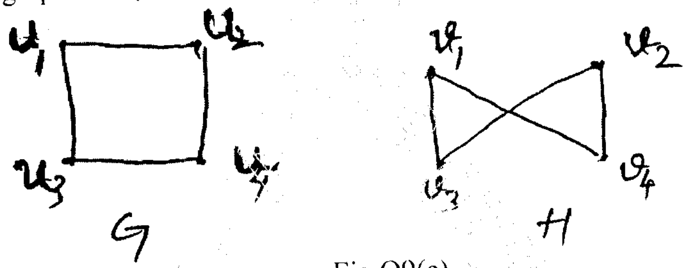

Fig.Q9(c)

OR

10 a. Find the cut vertices and cut edges in the graph, G. (Refer Fig. Q10(a)). **(04 Marks)**

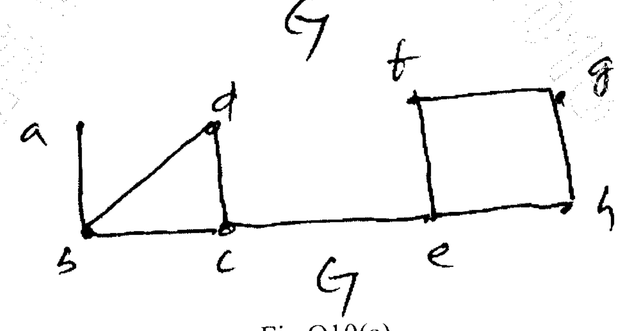

Fig.Q10(a)

b. Show that $K_{3,3}$ is non – planar. **(06 Marks)**

c. What is coloring? What is the chromatic number of the graphs K_n and $C_n (n \geq 3)$? **(06 Marks)**

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